Digital Modulation Basics
Outline

• Introduction to digital modulation
• Relevant modulation schemes
• Geometric representations
• Coherent & Non-Coherent Detection
• Modulation spectra
Modulation & Demodulation

- Carrier
  - Baseband Modulation
  - Data in
- Radio Channel
- Carrier
  - Synchronization/Detection/Decision
  - Data out
Modulation

• Modulation - process (or result of the process) of translation the baseband message signal to bandpass (modulated carrier) signal at frequencies that are very high compared to the baseband frequencies.

• Demodulation is the process of extracting the baseband message back the modulated carrier.

• An information-bearing signal is non-deterministic, i.e. it changes in an unpredictable manner.
Why Carrier?

• Effective radiation of EM waves requires antenna dimensions comparable with the wavelength:
  – Antenna for 3 kHz would be ~100 km long
  – Antenna for 3 GHz carrier is 10 cm long

• Sharing the access to the telecommunication channel resources
Modulation Process

\[ f = f(a_1, a_2, a_3, \ldots a_n, t) \]  (= carrier)
\[ a_1, a_2, a_3, \ldots a_n \] (= modulation parameters)
\[ t \] (= time)

- Modulation implies varying one or more characteristics (modulation parameters \(a_1, a_2, \ldots a_n\)) of a carrier \(f\) in accordance with the information-bearing (modulating) baseband signal.
- Sinusoidal waves, pulse train, square wave, etc. can be used as carriers
Continuous Carrier

Carrier: $A \sin[\omega t + \varphi]$
- $A = \text{const}$
- $\omega = \text{const}$
- $\varphi = \text{const}$

• Amplitude modulation (AM)
  - $A = A(t)$ – carries information
  - $\omega = \text{const}$
  - $\varphi = \text{const}$

• Frequency modulation (FM)
  - $A = \text{const}$
  - $\omega = \omega(t)$ – carries information
  - $\varphi = \text{const}$

• Phase modulation (PM)
  - $A = \text{const}$
  - $\omega = \text{const}$
  - $\varphi = \varphi(t)$ – carries information
Amplitude Shift Keying (ASK)

- Pulse shaping can be employed to remove spectral spreading
- ASK demonstrates poor performance, as it is heavily affected by noise, fading, and interference
Frequency Shift Keying (FSK)

Baseband
Data

BFSK modulated signal

where $f_0 = A \cos(\omega_c - \Delta \omega) t$ and $f_1 = A \cos(\omega_c + \Delta \omega) t$

- Example: The ITU-T V.21 modem standard uses FSK
- FSK can be expanded to a M-ary scheme, employing multiple frequencies as different states
Phase Shift Keying (PSK)

- Major drawback – rapid amplitude change between symbols due to phase discontinuity, which requires infinite bandwidth. Binary Phase Shift Keying (BPSK) demonstrates better performance than ASK and BFSK.
- BPSK can be expanded to a M-ary scheme, employing multiple phases and amplitudes as different states.

Baseband
Data

BPSK
modulated
signal

where $s_0 = -\text{Acos}(\omega_c t)$ and $s_1 = \text{Acos}(\omega_c t)$
Differential Modulation

• In the transmitter, each symbol is modulated relative to the previous symbol and modulating signal, for instance in BPSK \(0 = \text{no change}, 1 = +180^\circ\)

• In the receiver, the current symbol is demodulated using the previous symbol as a reference. The previous symbol serves as an estimate of the channel. A no-change condition causes the modulated signal to remain at the same 0 or 1 state of the previous symbol.
DPSK

- Differential modulation is theoretically 3dB poorer than coherent. This is because the differential system has 2 sources of error: a corrupted symbol, and a corrupted reference (the previous symbol).
- DPSK = Differential phase-shift keying: In the transmitter, each symbol is modulated relative to (a) the phase of the immediately preceding signal element and (b) the data being transmitted.
Pulse Carrier

• Carrier:
  A train of identical pulses regularly spaced in time
Pulse-Amplitude Modulation (PAM)

- Modulation in which the amplitude of pulses is varied in accordance with the modulating signal.
- Used e.g. in telephone switching equipment such as a private branch exchange (PBX)
Pulse-Duration Modulation (PDM)

Modulation in which the duration of pulses is varied in accordance with the modulating signal.

*Deprecated synonyms:* pulse-length modulation, pulse-width modulation.

Used e.g. in telephone switching equipment such as a private branch exchange (PBX)
Pulse-Position Modulation (PPM)

• Modulation in which the temporal positions of the pulses are varied in accordance with some characteristic of the modulating signal.
Ultra-Wideband (UWB) Systems

• Radio or wireless devices where the occupied bandwidth is greater than 25% of the center frequency or greater than 1.5 GHz.

• Radio or wireless systems that use narrow pulses (on the order of 1 to 10 nanoseconds), also called carrierless or impulse systems, for communications and sensing (short-range radar).

• Radio or wireless systems that use time-domain modulation methods (e.g., pulse-position modulation) for communications applications, or time-domain processing for sensing applications.
Demodulation & Detection

• Demodulation
  – Is process of removing the carrier signal to obtain the original signal waveform

• Detection – extracts the symbols from the waveform
  – Coherent detection
  – Non-coherent detection
Coherent Detection

- An estimate of the channel phase and attenuation is recovered. It is then possible to reproduce the transmitted signal and demodulate.
- Requires a replica carrier wave of the same frequency and phase at the receiver.
- The received signal and replica carrier are cross-correlated using information contained in their amplitudes and phases.
- Also known as synchronous detection
Coherent Detection 2

• Carrier recovery methods include
  – Pilot Tone (such as Transparent Tone in Band)
    • Less power in the information bearing signal, High peak-to-mean power ratio
  – Carrier recovery from the information signal
    • E.g. Costas loop

• Applicable to
  – Phase Shift Keying (PSK)
  – Frequency Shift Keying (FSK)
  – Amplitude Shift Keying (ASK)
Non-Coherent Detection

- Requires no reference wave; does not exploit phase reference information (envelope detection)
  - Differential Phase Shift Keying (DPSK)
  - Frequency Shift Keying (FSK)
  - Amplitude Shift Keying (ASK)
- Non coherent detection is less complex than coherent detection (easier to implement), but has worse performance.
Geometric Representation

• Digital modulation involves choosing a particular signal $s_i(t)$ from a finite set $S$ of possible signals.

• For binary modulation schemes a binary information bit is mapped directly to a signal and $S$ contains only 2 signals, representing 0 and 1.

• For M-ary keying $S$ contains more than 2 signals and each represents more than a single bit of information. With a signal set of size $M$, it is possible to transmit up to $\log_2 M$ bits per signal.
Geometric Representation 2

• Any element of set $S$ can be represented as a point in a vector space whose coordinates are basis signals $\phi_j(t)$ such that

\[
\int_{-\infty}^{\infty} \phi_i(t) \phi_j(t) dt = 0, i \neq j; \quad (= \text{are orthogonal})
\]

\[
E = \int_{-\infty}^{\infty} [\phi_i(t)]^2 dt = 1; \quad (= \text{normalization})
\]

Then

\[
s_i(t) = \sum_{j=1}^{N} s_{ij} \phi_j(t)
\]
Example: BPSK Constellation Diagram

\[
S_{BPSK} = \left\{ \left[ s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \right], \left[ s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \right] \right\}; \quad 0 \leq t \leq T_b
\]

\(E_b = \text{energy per bit}; \quad T_b = \text{bit period}\)

For this signal set, there is a single basic signal

\[
\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t); \quad 0 \leq t \leq T_b
\]

\[
S_{BPSK} = \left\{ \left[ \sqrt{E_b} \phi_1(t) \right], \left[ -\sqrt{E_b} \phi_1(t) \right] \right\}
\]
Constellation diagram

= graphical representation of the complex envelope of each possible symbol state
  – The x-axis represents the in-phase component and the y-axis the quadrature component of the complex envelope
  – The distance between signals on a constellation diagram relates to how different the modulation waveforms are and how easily a receiver can differentiate between them.
QPSK

• Quadrature Phase Shift Keying (QPSK) can be interpreted as two independent BPSK systems (one on the I-channel and one on Q), and thus the same performance but twice the bandwidth efficiency

• Large envelope variations occur due to abrupt phase transitions, thus requiring linear amplification
Quadrature Phase Shift Keying (QPSK) has twice the bandwidth efficiency of Binary Phase Shift Keying (BPSK) since 2 bits are transmitted in a single modulation symbol.

Carrier phases:
- QPSK: \{0, \pi/2, \pi, 3\pi/2\}
- BPSK: \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}
Eye Diagram

- Eye pattern is an oscilloscope display in which digital data signal from a receiver is repetitively superimposed on itself many times (sampled and applied to the vertical input, while the data rate is used to trigger the horizontal sweep).
- It is so called because the pattern looks like a series of eyes between a pair of rails.
- If the “eye” is not open at the sample point, errors will occur due to signal corruption.
Types of QPSK

• Conventional QPSK has transitions through zero (i.e. $180^0$ phase transition). Highly linear amplifiers required.
• In Offset QPSK, the phase transitions are limited to $90^0$, the transitions on the I and Q channels are staggered.
• In $\pi/4$ QPSK the set of constellation points are toggled each symbol, so transitions through zero cannot occur. This scheme produces the lowest envelope variations.
• All QPSK schemes require linear power amplifiers
Multi-level (M-ary) Phase and Amplitude Modulation

- Amplitude and phase shift keying can be combined to transmit several bits per symbol.
  - Often referred to as linear as they require linear amplification.
  - More bandwidth-efficient, but more susceptible to noise.
- For M=4, 16QAM has the largest distance between points, but requires very linear amplification. 16PSK has less stringent linearity requirements, but has less spacing between constellation points, and is therefore more affected by noise.
Distortions

Perfect channel

White noise

Phase jitter
GMSK

• Gaussian Minimum Shift Keying (GMSK) is a form of continuous-phase FSK in which the phase change is changed between symbols to provide a constant envelope. Consequently it is a popular alternative to QPSK

• The RF bandwidth is controlled by the Gaussian low-pass filter bandwidth. The degree of filtering is expressed by multiplying the filter 3dB bandwidth (B) by the bit period of the transmission (T), i.e. by BT

• GMSK allows efficient class C non-linear amplifiers to be used
Modulation Spectra

- The Nyquist bandwidth is the minimum bandwidth that can represent a signal (within an acceptable error).
- The spectrum occupied by a signal should be as close as practicable to that minimum, otherwise adjacent channel interference occur.
- The spectrum occupied by a signal can be reduced by application of filters.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Relative Magnitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nyquist Minimum Bandwidth</td>
<td></td>
</tr>
<tr>
<td>Adjacent Channel</td>
<td></td>
</tr>
</tbody>
</table>

29/01/2003
Property of R. Struzak
Bandwidth Efficiency

\[ \frac{f_b}{W} = \log_2 \left( 1 + \frac{E_b f_b}{\eta W} \right) \]

- \( f_b \) = capacity (bits per second)
- \( W \) = bandwidth of the modulating baseband signal (Hz)
- \( E_b \) = energy per bit
- \( \eta \) = noise power density (watts/Hz)

Thus

\[ E_b f_b = \text{total signal power} \]
\[ \eta W = \text{total noise power} \]
\[ \frac{f_b}{W} = \text{bandwidth use efficiency} \]

= bits per second per Hz
# Comparison of Modulation Types

<table>
<thead>
<tr>
<th>Modulation Format</th>
<th>Bandwidth efficiency C/B</th>
<th>Log2(C/B)</th>
<th>Error-free Eb/No</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 PSK</td>
<td>4</td>
<td>2</td>
<td>18dB</td>
</tr>
<tr>
<td>16 QAM</td>
<td>4</td>
<td>2</td>
<td>15dB</td>
</tr>
<tr>
<td>8 PSK</td>
<td>3</td>
<td>1.6</td>
<td>14.5dB</td>
</tr>
<tr>
<td>4 PSK</td>
<td>2</td>
<td>1</td>
<td>10dB</td>
</tr>
<tr>
<td>4 QAM</td>
<td>2</td>
<td>1</td>
<td>10dB</td>
</tr>
<tr>
<td>BFSK</td>
<td>1</td>
<td>0</td>
<td>13dB</td>
</tr>
<tr>
<td>BPSK</td>
<td>1</td>
<td>0</td>
<td>10.5dB</td>
</tr>
</tbody>
</table>
Spectral Efficiencies - Examples

- **GSM Digital Cellular**
  - Data Rate = 270kb/s; Bandwidth = 200kHz
  - Bandwidth efficiency = $\frac{270}{200} = 1.35$bits/sec/Hz

- **IS North American Digital Cellular**
  - Data Rate = 48kb/s; Bandwidth = 30kHz
  - Bandwidth efficiency = $\frac{48}{30} = 1.6$bits/sec/Hz
Modulation Summary

• Phase Shift Keying (PSK) is often used as it provides efficient use of RF spectrum. $\pi/4$ QPSK (Quadrature PSK) reduces the envelope variation of the signal.
• High level M-array schemes (such as 64-QAM) are very bandwidth-efficient but more susceptible to noise and require linear amplification
• Constant envelope schemes (such as GMSK) allow for non-linear power-efficient amplifiers
• Coherent reception provides better performance but requires a more complex receiver
References

• Campbell AT. *Untangling the Wireless Web – Radio Channel Issues*, Lecture Notes E6951, comet.columbia.edu/~campbell

• Fitton M. *Principles of Digital Modulation*, Lecture Notes ICTP 2002

• Proakis J. *Digital Communications*, McGraw & Hill Int.

• Rappaport TS. *Wireless Communications*, Prentice Hall PTR